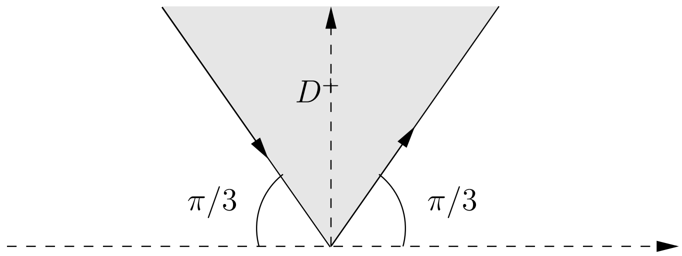
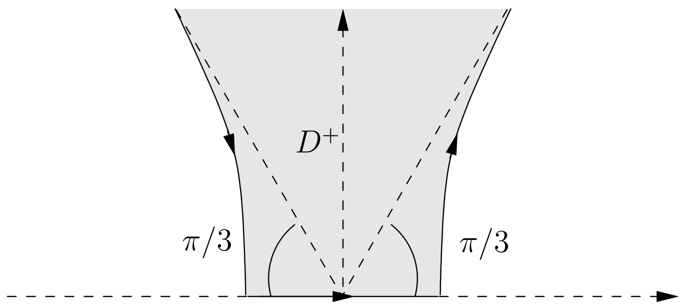


(a) The region D^+ for (4.1).



(b) The region D^+ for (1.1).

Fig. 4.1 *The regions* D^+ *for* (4.1) *and* (1.1).

4.1. Step I. Equation (4.1) can be written as the following one-parameter family of equations in divergence form:

$$\left[\mathrm{e}^{-\mathrm{i}\nu x-\mathrm{i}\nu^3 t}u\right]_t - \left[\mathrm{e}^{-\mathrm{i}\nu x-\mathrm{i}\nu^3 t}\left(\nu^2 u - \mathrm{i}\nu u_x - u_{xx}\right)\right]_x = 0, \quad \nu \in \mathbb{C}. \tag{4.5}$$

Indeed, the formal adjoint (defined in section 2.3) of the PDE (4.1) is

$$-v_t - v_{xxx} = 0 \tag{4.6}$$

and the divergence form is

$$(vu)_t + (v_{xx}u - v_x u_x + v u_{xx})_x = 0. \tag{4.7}$$

Separation of variables gives the one-parameter family of solutions to (4.6)

$$v = \mathrm{e}^{-\mathrm{i}\nu x-\mathrm{i}\nu^3 t}, \tag{4.8}$$

and thus we obtain (4.5). Note that we find v without specifying the domain or boundary conditions. Furthermore, in contrast to the classical transform method, we have *not* favored either the x- or the t-variable (hence the "synthesis" as opposed to "separation").

Suppose that the PDE (4.1) is valid in the domain $\{0 < \xi < \infty, 0 < \tau < t\}$. Then, applying Green's theorem in this domain, (4.5) implies the following global relation:

$$\mathrm{e}^{-\mathrm{i}\nu^3 t}\hat{u}(\nu, t) = \hat{u}_0(\nu) - \tilde{g}(\nu, t), \quad \Im\nu \le 0, \quad 0 < t < T, \tag{4.9}$$