Nonlinear dynamics of a bistable piezoelectric-composite energy harvester for broadband application

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Abstract

The continuing need for reduced power requirements for small electronic components, such as wireless sensor networks, has prompted renewed interest in recent years for energy harvesting technologies capable of capturing energy from ambient vibrations. A particular focus has been placed on piezoelectric materials and devices due to the simplicity of the mechanical to electrical energy conversion and their high strain energy densities compared to electrostatic and electromagnetic equivalents. In this paper an arrangement of piezoelectric layers attached to a bistable asymmetric laminate is investigated experimentally to understand the dynamic response of the structure and power generation characteristics. The inherent bistability of the underlying structure is exploited for energy harvesting since a transition from one stable configuration to another, or 'snap-through', is used to repeatedly strain the surface bonded piezoelectric and generate electrical energy. This approach has been shown to exhibit high levels of power extraction over a wide range of vibrational frequencies. Using high speed digital image correlation, a variety of dynamic modes of oscillation are identified in the harvester. The sensitivity of such modes to changes in vibration frequency and amplitude are investigated. Power outputs are measured for repeatable snap-through events of the device and are correlated with the measured modes of oscillation. The typical power generated is approximately 3.2mW, comparing well with the needs of typical wireless senor node applications.

1. Introduction

Bistable composite laminates have been studied extensively in recent years, primarily with the focus on their use for aerospace morphing and shape changing applications [1]. Early work by Hyer [2] observed that under certain geometric conditions a carbon fibre reinforced plastic composite (CFRP) laminate with an asymmetric stacking sequence can exhibit two differently curved stable configurations. For morphing applications this is desirable as a large structural deformation can be achieved by actuating the laminate between its two configurations through an instantaneous energy input, often via an electrical input to a smart actuator. In the work presented here the intention is to reverse the process and use the bistable piezoelectric-laminate combination for energy harvesting applications; mechanical vibrations lead to 'snap-through' of the bistable laminate between its stable states to generate piezo-electricity.

With the recent increase in the use of wireless sensor networks and electronics requiring a portable energy source, energy harvesting devices have been developed in an attempt to convert ambient vibrations to electrical energy via electrostatic generation [3], electromagnetic induction [4], and the piezoelectric effect [5]. Priya [6] demonstrated that piezoelectrics have several advantages, including ease of integration within a system, higher strain energy densities than electrostatic and electromagnetic systems, and the simplicity of converting strain energy to electrical energy. Figure 1 shows a schematic representation of the piezoelectric energy harvesting concept. The intention is to harvest electrical energy from waste mechanical energy in the form of ambient vibrations, which generates an alternating voltage using a piezoelectric material. For practical applications the alternating voltage that is generated is converted to a stable rectified voltage through an AC-DC

converter and is stored so that it can be subsequently used to power a low-power device, such as a wireless sensor network for structural health monitoring.
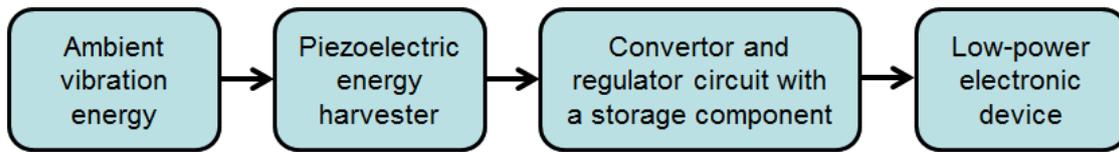

Figure 1: Piezoelectric energy harvesting concept for mechanical vibration to electrical energy conversion.

Vibration based harvesting devices are often tuned to operate near resonance to maximise their power generation, however, such devices are not easily scalable and their harvesting performance falls significantly away from these frequencies [7]. This often renders linear resonant systems unsuitable where vibrations exhibit multiple time-dependent frequencies, such as the vibration patterns induced by railway carriages [8], automobiles [9] or aircraft [10]. The work here considers a bistable laminate since it was recently shown [5] that a nonlinear $[0_2/90_2]$ bistable composite with bonded piezoelectric layers was effective at harvesting energy over a wide range of frequencies.

1.1 Bistable Asymmetric Laminates

When the individual plies of a composite laminate are stacked in an asymmetric stacking sequence the resulting mismatch in thermal properties between plies results in thermally induced residual stresses. This ultimately leads to a curved deformation of the laminate. For the vast majority of composite applications, this warping due to the anisotropic thermal response to elevated manufacturing temperatures is considered undesirable and is typically avoided by ensuring that the laminate layup is both balanced and symmetric. However, under certain geometric conditions the thermal warping of an asymmetric laminate can lead to two distinct stable states of different curvature, resulting in a large achievable structural deformation from a relatively small mechanical strain input from a smart actuator or ambient vibrations. Figure 2 shows this behaviour for an example laminate with a [0/90] stacking sequence, as considered in Arrieta et al. [5]. For a low ratio of edge length to thickness (approximately ≤ 80) only a single stable solution exists with an anticlastic, or 'saddle' shape, point A. As the geometric ratio increases the solution bifurcates, point B. Beyond this point two approximately cylindrical stable shapes are observed, points C and D, while the saddle shape becomes unstable. As the ratio increases further the difference in curvature between the two stable states increases, resulting in a larger structural deformation upon mechanical deformation between states.

For energy harvesting applications additional layers of piezoelectric material are attached to the surfaces of the bistable laminate. Any structural deformation between the cylindrical configurations, from point C to point D, will strain the piezoelectric material and generate an electrical voltage. When the piezoelectric-laminate combination is exposed to mechanical vibrations which are sufficient to induce snap-through between states, the piezoelectric material will be repeatedly strained to generate a potentially useful power output.
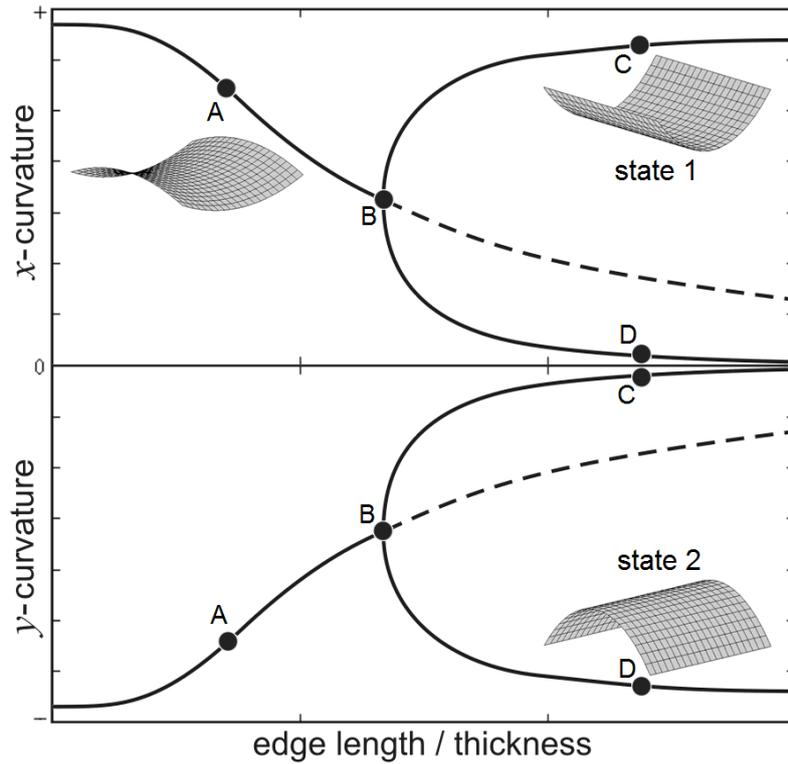
Figure 2: Stable (solid) and unstable (dashed) shapes of a [0/90] laminate.

Analytical modelling of the thermally-induced shapes of asymmetric composite laminates has received a great deal of interest in the past few decades [1, 2, 11-21]. Since the vast majority of this research focused on the use of bistable laminates for structural morphing (shape change), modelling has been largely limited to the prediction of static shapes and strain or the energy input necessary for actuation and 'snap-through'. The dynamic response of such structures to mechanical vibrations or dynamic loads has been largely neglected. Refinements to the initial work on analytical modelling of the static shapes of bistable asymmetric laminates [11-13] have included the inclusion of smart actuators [14-15] and an increase in order of the polynomials used to characterise the doubly curved shapes [16]. These static models have been applied to optimisation of bistable vibration energy harvesters [17-19] that omit the snap-through process. The snap-through event for a bistable laminate is a highly nonlinear process and less research has been conducted in this area. Arrieta et al. [20] studied the dynamic response of a bistable laminate to mechanical vibrations by considering the low amplitude nonlinear oscillations about each of the stable shapes (i.e. no snap-through event). These results were compared to a low order model showing excellent agreement in amplitude of response. Diaconu et al. [21] presented an analytical model for the dynamic response of a single snap-through event due to concentrated forces, introducing a time-dependent extension to the classic Rayleigh-Ritz strain energy minimisation principles employed in earlier static modelling work [13]. Due to the simplifications in the analytical model, notably a constant curvature approximation, the snap-through loads were found to be 30-50% higher than those predicted by finite element modelling.

With the development of bistable laminates for emerging energy harvesting applications there is an increasing need to understand the dynamics of such a system using experimental characterisation. There is also a need for new modelling approaches capable of predicting the time response of the structure, and the associated voltage response subject to variable vibrations. An important first step is to characterise this response experimentally to inform the development of dynamic modelling which can subsequently be used as a design tool for maximised energy harvesting capability. This paper aims to deepen the understanding of the dynamic behaviour of thin bistable composite plates subjected to mechanical vibrations. In particular, this work is intended to characterise the differing

modes of oscillations due to mechanical vibrations with a view to informing the design of broadband bistable vibration energy harvesters.

2. Experimental Methods

This experimental study characterises the dynamic response of bistable piezoelectric-composite plates subject to mechanical vibrations of differing frequency in terms of the peak-to-peak displacement and power harvested.

2.1. Device Definition and Test Setup

A square [0/90] laminate is considered as the basis for developing a broadband energy harvesting device. The laminate measures 190×190mm and is made from M21/T800 CFRP prepreg material [22]. A single piezoelectric Macro Fiber Composite (MFC) layer (M8585-P2 [23], 85×85mm) is attached to the laminate surface. The device is polarised through thickness so that the direct d_{31} piezoelectric effect is aligned with the major curvature of the dominant shape (see Fig. 3a, state 2). Such a device configuration also exhibits a high capacitance (typically 800nF) compared to devices polarised along their length. Additional masses in the form of steel bolts (~12g each) are attached to the four corners of the laminate to increase the achievable curvatures and aid snap-through during oscillation using a mechanical shaker. When considering optimisation of the device design for maximised power output, the size and location of the distributed masses represent an additional parameter which may be optimised for specific practical applications. The device is mounted to the shaker from its centre, with a 15mm circular section carefully removed from the MFC to allow attachment, Fig. 3b. The MFC is attached to an oscillo-scope to measure open-circuit voltage outputs during testing.
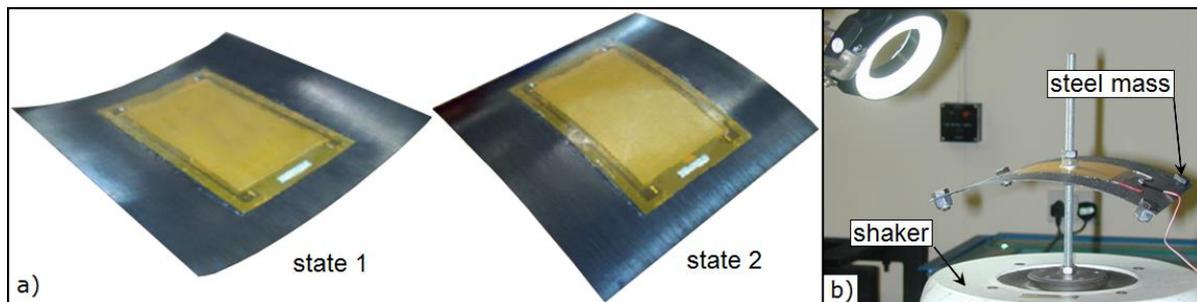

Figure 3: a) Stable shapes of a [0/90] piezoelectric plate, and b) experimental setup showing mechanical shaker attachment.

2.2. Displacement Capture

The three-dimensional displacements of the harvesting device subjected to mechanical vibrations are captured via a Digital Image Correlation (DIC) system to correlate the mode of vibration with the power generated. This system uses two high speed cameras in stereo to take images of a painted speckled pattern (Fig. 4a) on the laminate surface at 5000 frames per second. At each time interval the images are compared against a reference image and the movement of the speckled pattern tracked to produce a map of in-plane and out-of-plane displacements. An example plot of the out-of-plane displacement of the laminate in one cylindrical configuration is shown in Fig. 4b, where the red contour indicates a high displacement and the purple contour denotes zero displacements. Note that the pattern is disrupted around the center attachment and the four corner masses due to the difficulty in tracking the associated geometries and these areas only being visible in one of the two camera images.
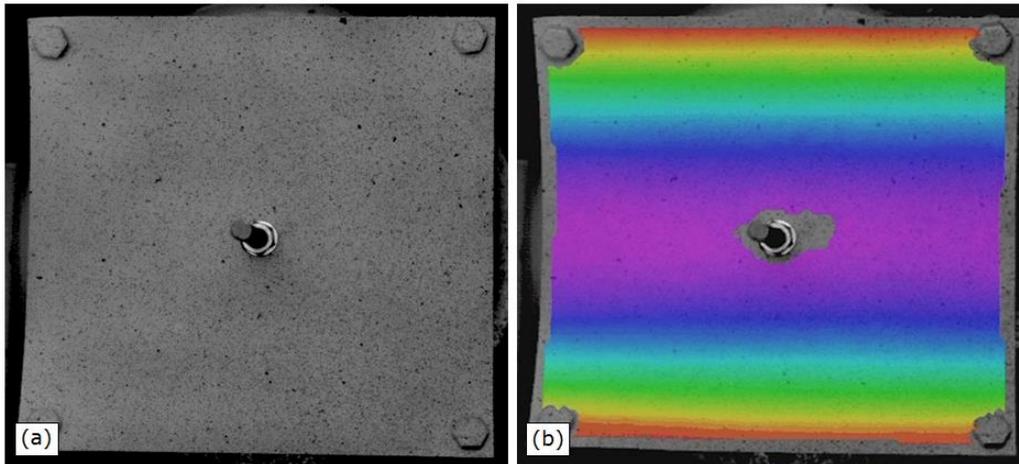
Figure 4: a) Digital Image Correlation reference frame showing speckled surface pattern on the 190mm × 190mm laminate, and b) out-of-plane displacement plot for one cylindrical configuration. Note: Purple denotes zero displacement, red denotes maximum displacement (12mm).

3. Modes of Oscillation

Ambient vibrations, such as those experienced in transport [8-10], are rarely periodic. Instead, vibration patterns may exhibit multiple time-dependent frequencies of widely varying amplitude. In developing a broadband energy harvester to generate an electrical output across a range of vibrational frequencies, an understanding of the varied dynamic responses is essential. The DIC displacement capture technique is used to study the different modes of the bistable device under a range of vibrational inputs in terms of both frequency and displacement from the shaker.

Sufficient levels of acceleration lead to snap-through across a wide range of input frequencies. For lower acceleration levels, snap-through is more readily achieved at frequencies close to resonance. In this section we consider both a low acceleration, resonant frequency of 18Hz to demonstrate repeatable snap-through, and an off-resonance frequency of 20.8Hz to demonstrate alternative modes. These different modes are identified through variation in the peak-to-peak displacement at constant frequency.

Initially we consider the low amplitude oscillations resulting from excitation at the off-resonance frequency of 20.8Hz with a peak-to-peak centre displacement of 6mm. Figure 5a shows a phase plot of the response where the displacement near one corner of the laminate, selected from the DIC data, is plotted against the velocity at the centre which is a measure of the laminate position within each oscillation. A repeatable cycle is observed where the corner displacement remains positive, indicating oscillations about one state only, i.e. there are no snap-through events for this condition. It is also noted that the maximum displacements are observed at zero centre velocity, indicating that the corner and centre of the laminate displace in phase with one another. Increasing the vibrations to 7mm center displacement at the same frequency (20.8Hz) reveals a less predictable response with sporadic and chaotic snap-through events, Fig. 5b, demonstrating that snap-through is achievable in an off-resonance condition. In this particular state it is observed that when snap-through does occur it generally originates at one specific corner of the device, possibly due to the presence of a small degree of asymmetry introduced during manufacture of the piezoelectric-laminate combination.
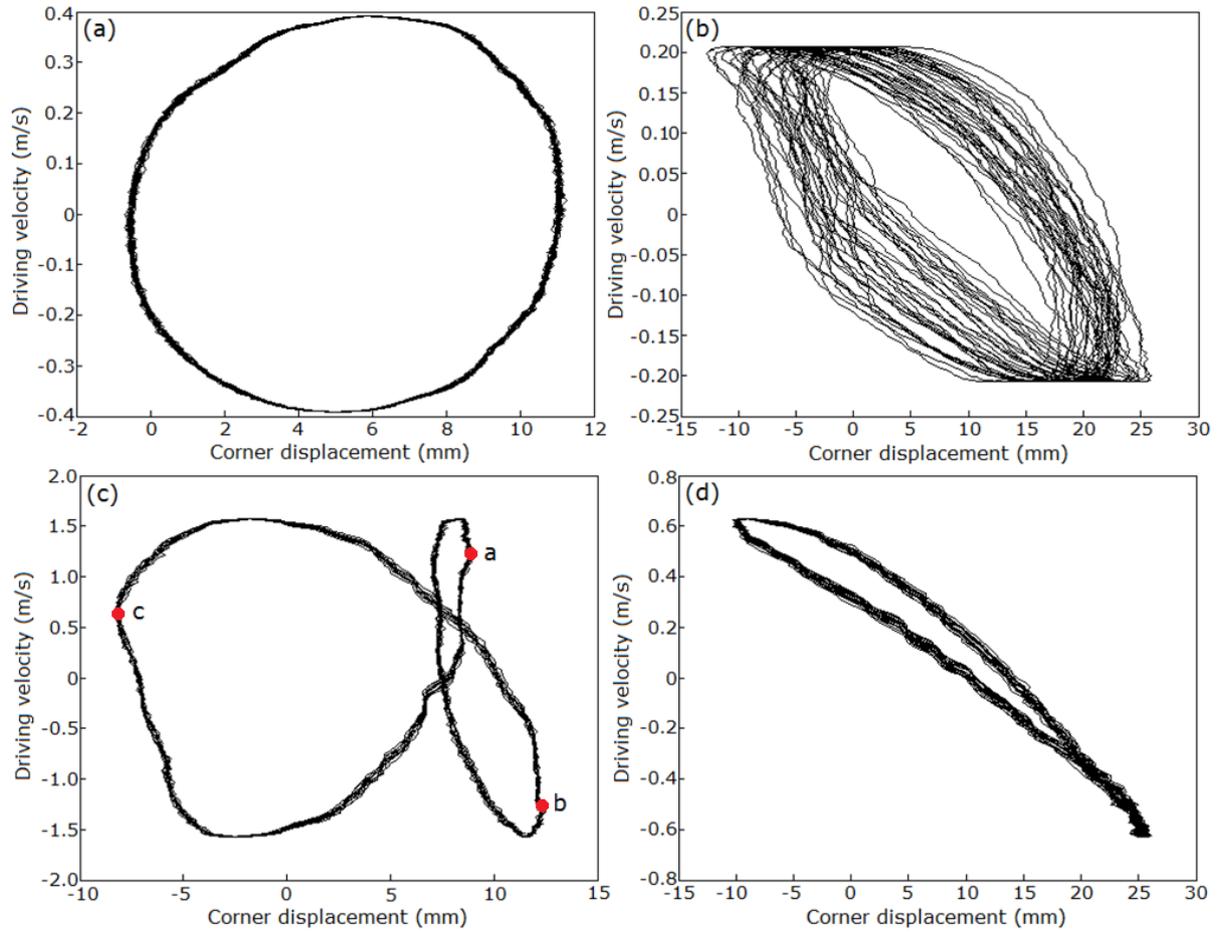
Figure 5: Phase plots showing a) low amplitude oscillations and no snap-through (20.8Hz, 6mm peak-to-peak displacement), b) non-uniform response and chaotic snap-through (20.8Hz, 7mm), c) intermittent snap-through (20.8Hz, 9mm), and d) repeatable snap-through (18Hz, 3.5mm).

Increasing the centre displacement to 9mm at the same frequency removes the chaotic nature of the response as the device settles into a pattern of intermittent but repeatable snap-through events. Figure 5c shows this phase plot with a large negative displacement at one extreme, and a double loop in the positive region. To better understand this particular response, Fig. 6 shows three individual DIC frames selected from this phase plot, showing contour plots of the out-of-plane displacements. The displacements of Figs. 6a-c correspond to the top left corner displacement of the laminate at points a, b and c respectively in Fig. 5c.

Figure 6a shows the device in a deformed state where the lower half of the laminate has snapped to the opposite cylindrical shape while the top half remains in the first cylindrical shape. The upper corner displacement is therefore smaller than would be expected for a full transition between states (see point 'a' in Fig 5c). Figure 6b shows the expected dominant shape with approximately cylindrical configuration as in Fig 3a and corresponds to point 'b' in Fig 5c. Finally Fig. 6c shows that the upper half of the laminate has now snapped, while the bottom half remains in the dominant cylindrical configuration (point 'c' in Fig 5c). This pattern repeats consistently for all subsequent oscillations, and is indistinguishable from uniform and complete snap-through to the naked eye. If we consider the strain which is induced in the piezoelectric material under these conditions, the result will be reduced from that associated with complete snap-through of the laminate. Thus, identifying these lower modes of oscillations is critical in prediction of power outputs and improved device design.
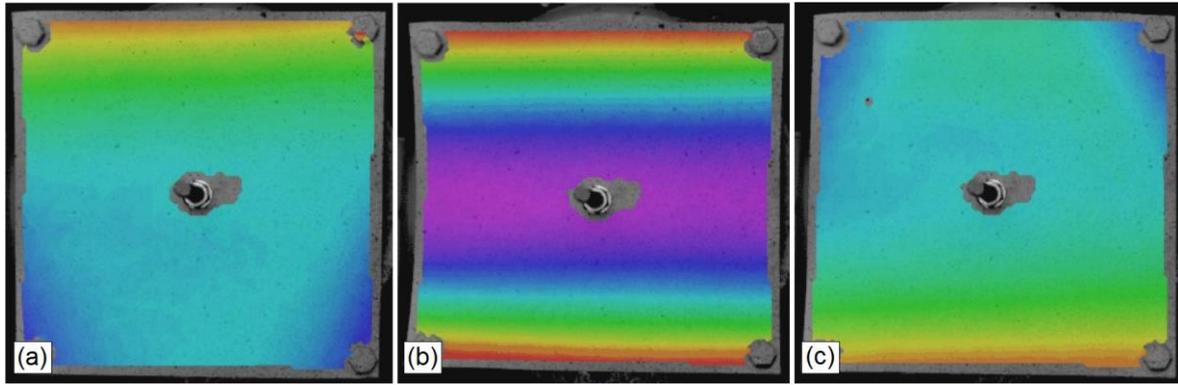
Figure 6: Three frames captured using the DIC system for the intermittent snap-through pattern seen in Fig. 5c. a) Bottom half of the laminate snaps only, b) full transition to the dominant cylindrical configuration, c) top half of the laminate snaps only. Figures a-c correspond to the displacements at points a-c respectively in Fig. 5c. Note: (a and c) Red denotes positive displacement, blue negative displacement, (b) red denotes positive displacement, purple zero displacement.

Having demonstrated the potential for off-resonance snap-through events, repeatable snap-through at every oscillation is demonstrated close to the resonant frequency at 18Hz, achievable with a lower centre displacement of 3.5mm as in Fig. 5d. Both large negative and positive displacements are observed for every oscillation due to the repeated snap-through events; interestingly these peak values do not occur at zero centre velocity suggesting that the centre and corner displacements are out of phase from one another.

The peak-to peak displacements used in these tests range from 3.5mm for 18Hz to 9mm for 20.8Hz, corresponding to peak accelerations experienced by the energy harvester of 2.3 – 7.8g. However, it has previously been shown [17-18] that the dimensions of the device, and thus the necessary g level to induce the different oscillatory modes, are scalable. Device dimensions are therefore tunable to suit a wide range of vibration sources. In the current configuration, state 2 (Fig. 3a) is the lowest energy state due to the alignment of the piezoelectric layer, and is therefore the dominant static shape. Snap-through to state 1, and vice versa, are more easily achieved close to the resonant frequency, where the band of frequencies enabling snap-through widens with both an increase in the vibration amplitude, or through scaling (reduction) in the device dimensions.

Clearly the power outputs of the device as an energy harvester will be highly dependent on the different modes of oscillation identified. While the results above have examined the modes of oscillation the next section will discuss the power generated by the piezoelectric element during oscillation.

4. Energy Harvesting and Power Generation

The corresponding piezoelectric voltage outputs associated with each of the phase plots of Fig. 5 are shown in Fig. 7. For low amplitude oscillations (Figs. 5a and 7a) it is noted that the voltage output takes the form of a smooth sinusoidal wave with peak voltages of ±3V. While this value is relatively small, it demonstrates that snap-through is not a prerequisite to produce some level of power output. For the nonuniform response of Fig. 5b much larger peak voltages (up to 50V) are observed (Fig. 7b) at apparently arbitrary times, corresponding to the chaotic nature of the snap-through events. While this results in occasional higher voltage outputs the majority of the oscillations generate 10-20V. These fluctuations could be a concern when trying to provide a consistent power supply, although the effect can be smoothed by the AC-DC converter stage, as in Fig. 1. When the response settles to a predictable pattern of intermittent snap-through (Figs. 5c and 7c) the voltage output becomes more consistent, where each large peak (~40V) in Fig. 7c corresponds to the configuration seen in Fig. 6b.

Finally, for the repeatable snap-through pattern of Fig. 5d the voltage output, and therefore power, returns to a periodic shape, Fig. 7d. In this case the peak voltages are ±21V.
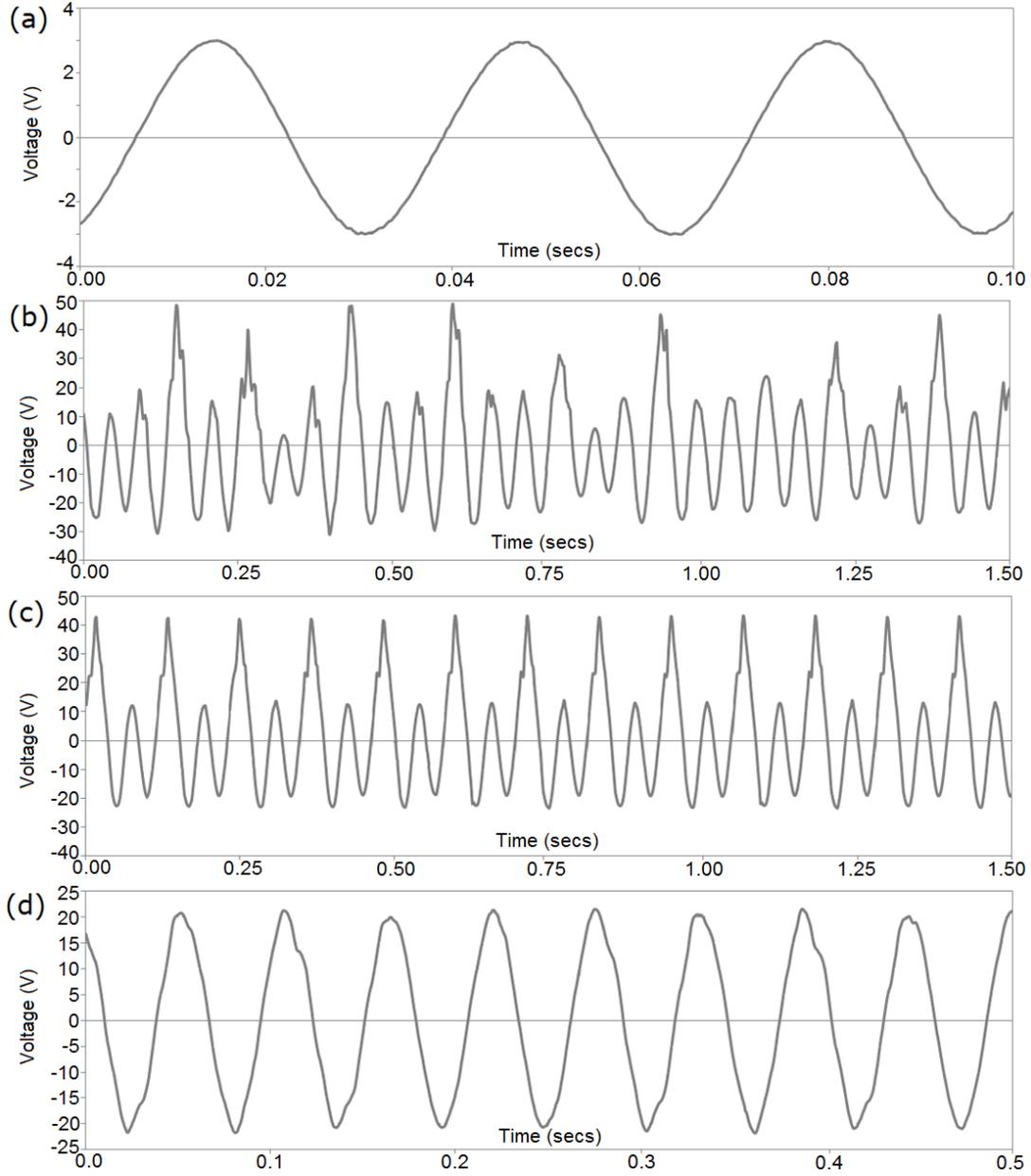

Figure 7: Voltage outputs associated with a) low amplitude oscillations, b) nonuniform behaviour, c) intermittent snap-through, and d) repeatable snap-through, corresponding to Figs. 5a-d respectively.

To consider the power energy harvesting capability of the bistable piezoelectric-composite laminate combination, we select the repeatable snap-through mode (Fig. 5d). Taking an experimentally measured capacitance C for the piezoelectric layer of 800nF, and an rms voltage V_{rms} of 14.9V (Fig. 7d) the electrical energy (E) associated with each actuation (twice per full oscillation) is 0.09mJ (Eq. 1), giving a power output (P) of 3.2mW for a frequency f of 18Hz (Eq. 2).

$$E = \frac{1}{2}CV_{rms}^2 \qquad (1)$$

$$P = Ef \qquad (2)$$

Applications for wireless sensor networking to date have focused on environmental monitoring where nodes tend to have low-power sleep states (10-300µW) with higher power requirements for transmitting and receiving data (500µW-60mW) [24]. The example power output of 3.2mW represents an arbitrarily chosen device geometry. However this value compares favourably with these typical power requirements.

5. Conclusion

This paper presents an experimental study of a bistable piezoelectric-composite plate for mechanical vibration to electrical energy harvesting. The arrangement of the device utilises the inherent bistability present in a specific family of asymmetric composite laminates to strain a piezoelectric layer bonded to the laminate. In the presence of mechanical vibrations 'snap-through' of the laminate between the two stable states leads to a large structural displacement and the generation of piezo-electricity. Through high-speed video imaging, this experimental study has revealed that the modes of oscillation are sensitive to the frequency and amplitude of the mechanical vibrations, with behavioural patterns of (i) small amplitude oscillation, (ii) uniform and nonuniform intermittent snap-through, and (iii) repeatable snap-through. Power outputs are largest when snap-through occurs, with generated power of the order of 3.2mW. This value is observed for an arbitrarily chosen device geometry and compares favourably with typical wireless sensor network applications [24]. This suggests potential and scope for improved power output through optimisation of the device design in terms of laminate lay-up, laminate topology and piezoelectric material and position. By further developing the understanding of the dynamic response of thin composite plates to mechanical vibrations this paper provides useful information for the development of novel dynamic models to enable design of vibration based energy harvesters for maximised power output.

Acknowledgements

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