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Distribution Network Pricing for Uncertain Load Growth using Fuzzy Set Theory

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Abstract—**The decarbonisation of transport and heating will introduce uncertain smart appliance growth in the power system, which fundamentally challenges traditional network pricing. In this paper, a new long-term distribution network charging is proposed to accommodate uncertain load growth. Instead of using fixed a load growth rate (LGR), it adopts a fuzzy model, developed based on a set of projected deterministic LGRs and confidence levels. This fuzzy model is incorporated into the pricing model through α–cut intervals. In order to improve computational efficiency, an analytical pricing approach is introduced. The Vertex Extension approach is used to build charge membership functions. Thereafter, a common defuzzification approach, Centre of Gravity (COG), is employed to defuzzify membership functions in order to generate deterministic charges. The new approach is benchmarked with two existing standard charging methods on a practical UK high-voltage distribution system. Results show that it is effective in capturing the uncertainty in load growth.**

Index Terms-- **Network pricing, load growth, fuzzy set, defuzzification, centre of gravity, distribution network.**

I. INTRODUCTION

MANY countries in the world are committed to reducing greenhouse gas emissions. The UK government is also among them, whose ambition is to reduce CO_2 by 80% by 2050 relative to the 1990 level. It has adopted a series of initiatives to improve energy efficiency and encouraged customer's interaction with energy networks, etc. in order to reduce the cost of transition to a low carbon economy.

Assisting the target, one effective way is through improving energy efficiency and electrifying traditional transport and heating. For example, in the UK, Electric Vehicles (EV) and Heat Pumps (HPs) demand would rise to 13GW in total by 2050 out of 123GW total demand [1]. Apart from benefits, the new appliances, together with traditional demand growth, could lead to unevenly distributed load growth across networks. The load growth may have already been saturated in some regions, less likely to have huge variations over the long term. By contrast, in regions with high economic progress and more customer willingness of switching to low-carbon demand, there might be more

plausible projections for long-term load growth. The National Grid in the UK stated that the electricity energy met via Western Power Distribution (WPD) network increased to 15 TWh by 2013-14, implying an average LGR of 1.4% per annum; the electricity met via Scottish Power (SP) network increased to 18 TWh by 2013-14, producing an average LGR of 0.6% per annum [2] . The uneven load growth would trigger substantial network investment if not properly managed.

One effective way for network operators to influence customers' decision in using networks is sending economic signals to them, in term of network charges, to affect their connecting sites and sizes [3-6]. Paper [7] extends the concept of locational marginal pricing (LMP) to distribution networks by formulating a distribution LMP to encourage the utilisation of energy storage. Paper [8] presents an integrated distribution LMP method to alleviate network congestion caused by increasing demand through using Electric Vehicles (EVs). In [9], the authors propose nodal pricing in distribution networks to reward DGs for reducing line losses. Paper [10] investigates the potential of implementing locational pricing to facilitate a complete change of current tariff methodologies. An extensive review of network pricing is reported in [11]. The approaches in these papers mainly focus on network operation and the uncertainties in future load growth are not considered. Other methodologies in [12, 13] only passively react to a set of projected patterns of demand/generation rather than proactively influence them [14].

One recent development in network pricing is the Long-run Incremental Cost Pricing (LRIC) [15, 16]. Because of its advantages of generating locational charges to respect both network utilisation and power travelling distance, it has been implemented on the EHV distribution networks of Western Power Distribution, UK Power Networks, and North Power Grid. Thus, the core of this LRIC model is inherited by the charging model proposed in this paper [12]. This LRIC, however, requires a peak load growth rate (LGR) to model future peak load growth, which is an averaged index and assumed to be the same during one price control period, normally 1.0% [2]. However, due to increasing smart appliances, a deterministic LGR for a large area is no longer representative enough. The UK's electricity and gas regulator-Office of Gas and Electricity Markets (Ofgem) also requires cost-reflectivity of distribution network charges [4].

Long term peak load forecasting is difficult due to lack of data and the accuracy is affected by many factors. The UK National Grid's Electricity Ten Year Statement [17] states that

peak demand forecasting is influenced by "projections for economic assumptions, energy efficiency measures, EVs, heat pumps and the impact of time-of-use tariffs". Thus, it employs four scenarios, No Progression, Low Progression, Low Carbon Life and Gone Green, to represent UK peak demand projections. Both fuzzy set approaches and probabilistic approaches exist in the literature for long-term peak load growth and uncertainty modelling [18]. Papers [19] [20] introduce combined fuzzy logic, regression method and artificial neural network (ANN) for load forecasting with the relationship between humidity, temperature and load considered. In paper [21], the authors introduce a new fuzzy logic method for mid-term energy forecasting, which transforms the input variables into differences. [22] proposes a probabilistic approach which can make use of hourly information to generate more accurate and defensible load forecasts. A model that provides probabilistic forecasting of magnitude and time of peak demand for one-year ahead is introduced in [23].

Probabilistic models describe randomness of variables which obey some type of probability distributions. By contrast, fuzzy set models describe vagueness that does not follow any probability distributions or is hard to model [24]. If there is sufficient information to build accurate probabilistic models, probabilistic methodologies are better than fuzzy set methods. On the contrary, fuzzy set methods are more efficient if little information is available [25]. Paper [24] uses a hybrid method of fuzzy set and Monte Carlo simulation for power system risk analysis, where both peak load and component failures are modeled as fuzzy membership functions. Long-term LGR falls into the second category, as it has a high degree of uncertainties due to lack of sufficient knowledge [26]. Although forecasting by nature is stochastic, most network operators today still develop and use point forecasting due to low-resolution data and a limited number of observations [22]. They only have a vague description of LGR and the confidence levels, but not enough statistical information to model its distributions. Therefore, it is more rigorous to model LGR with fuzzy set approaches, as also argued in [24] that fuzzy set approaches are appropriate for modelling uncertainty in peak load forecasting.

In this paper, a fuzzy LGR model is proposed to represent uncertain future peak load growth and a new analytical charging methodology is designed to include it. The method chooses a certain number of α–cut of LGR to calculate corresponding charges. For LGRs deviating from chosen α–cut intervals, their charges are evaluated according to the charges of these nearest α–cut intervals. The charges under different α–cut intervals for a branch are used to build its charge membership function with Vertex Extension. Then, Center of Gravity (COG) defuzzification method is employed to defuzzify charge membership functions. The new approach is compared with other two standard approaches on a notional two-bus system and a practical EHV distribution network.

The key contribution of this paper is to use fuzzy set theory to resolve a new challenging problem in power network

pricing, i.e. the uncertain future load growth. It is achieved by two key innovations: i) to extend the original LRIC to incorporate fuzzy LGR and respect uncertainties in peak load growth by building a new fuzzy load growth rate model; ii) to introduce a new analytical approach to calculate charges in order to easily accommodate the new fuzzy LGR.

The remainder of this paper is organized as follows: Section II provides a brief introduction to the original LRIC, introduces the fuzzy LGR model, and investigates the shape of charge functions with LGR. Sections III-V introduce the fuzzy charging model, defuzzificaiton of charge membership function and overall implementation steps of the new approach. Two demonstration examples are provided in Section VI. Section VII provides some practical considerations and Section VIII concludes this paper.

II. LONG-RUN INCREMENTAL COST PRICING AND FUZZY LOAD GROWTH RATE

This section introduces the original LRIC model and the modeling of fuzzy LGR. It finally investigates how the shape of LRIC charging functions changes with respect to LGR and network utilisation, which is key in mapping fuzzy LGRs into charging function.

A. Long-run Incremental Cost Pricing

LRIC charging reflects asset costs to meet a nodal increment by examining its impact on future investment [13] [27]. For a given LGR r, the investment horizon is the time taking load to grow from the current level to full loading level

$$C = D \cdot (1+r)^n \qquad (1)$$

where, n is time horizon, C is the capacity of a component, and D is current loading level.

Rearranging (1) and taking the logarithm of it gives

$$n = \frac{\log C - \log D}{\log(1+r)} \qquad (2)$$

With a new nodal increment Δp, the new time horizon, n_{new} can be obtained by replacing $\log D$ with $\log(D+\Delta p)$ in (2).

The present value (PV) of a component is

$$PV = \frac{AssetCost}{(1+d)^n} \qquad (3)$$

The change in PV with a discount rate d is

$$g(r) = \Delta PV = AssetCost \cdot \left(\frac{1}{(1+d)^{n_{new}}} - \frac{1}{(1+d)^n} \right) \qquad (4)$$

The LRIC price for one component is the annuitized change in its PV caused by the new injection/withdrawal. The LRIC charge for the increment is the summation of the annuitized ΔPV over all circuits.

$$LRIC_N = \frac{\sum_l \Delta PV}{\Delta P} \cdot AnnuityFactor \qquad (5)$$

where, l is the number of components supporting nodal ΔP.

While for large-size DGs, normally a negative injection Δp in the model is used and other procedures are the same as those for pricing demand.

B. Fuzzy Load Growth Rate

Fuzzy set theory, as a means to model the uncertainty described with vague linguistics, can incorporate more projected LGRs of various confidence levels. Thus, a fuzzy LGR can be built according to the deterministic values and confidence levels. One popular model is the triangular function in Fig.1.a. It reflects the proposition "*load growth is around r*". Here, *r* is the centre of the triangle with a confidence level of 1, the most likely LGR. r_1 and r_2 are the boundaries with the confidence level of 0. This model indicates that the LGR is likely to appear in areas $[r_1, r]$ and $[r, r_2]$ but with lower probability at far ends. The trapezoid models LGR described with the proposition "*LGR might be in [r1, r4] and the confidence varies from 0 to 1*". As given in Fig.1.b, the LGR may be anywhere in $[r_1, r_4]$, but more likely to appear in $[r_2, r_3]$ with a confidence level of 1 and less likely to occur in $[r_1, r_2]$ and $[r_3, r_4]$. The LGR definitely does not appear outside $[r_1, r_4]$, where the confidence level is zero.
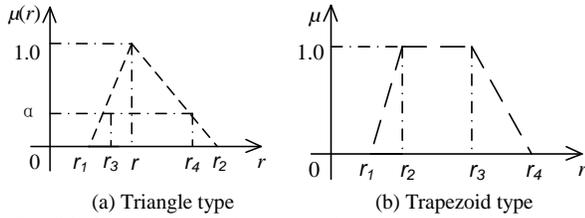

(a) Triangle type   (b) Trapezoid type
Fig.1. Possible fuzzy load growth rate models

The building of membership functions in practice actually depends on LGR's description and corresponding confidence levels. Normally, the information needed for describing future load growth can be obtained from local network operators' Long-Term Development Statements (LTDS) and national demand prediction. The fuzzy LGR will be mapped into charging model through α–cut [28].

The membership function of fuzzy LGR is built according to empirical descriptions and future load and generation predictions of network planners. It assumes that each projection has a relevant confidence level and network planners have knowledge of the LGR range based on their knowledge. The building involves the following steps:

i) Network planners project future peak demand and generation growth in their territories;

ii) A couple of projections are obtained, each with relevant confidence level;

iii) The projected small-size generation growth in each projection is deducted from demand growth;

iv) The most probable projection is set as the peak for the fuzzy LGR membership with a confidence level of 1.0, i.e. the Core (when α=1)

v) The highest and lowest projections, which are very unlikely to happen according to planners experience, are set as membership boundaries with zero confidence level, i.e. the Support (when α=0);

vi) Those projections with confidence level between 0 and 1.0 form the inflection points of the membership.

C. Shape of Charge Membership Function

It is necessary to determine whether the charge function is convex or concave with regard to a LGR before mapping the fuzzy LGR into LRIC. It is because that the maximum values of membership functions, which determine the shapes [28], have to be included in the Vertex Extension approach in building charge membership functions.

Taking the derivative of (3) with respect to *D* produces the sensitivity of branch's PV with respect to load change

$$\frac{\partial PV}{\partial D} = \frac{Asset}{C} \cdot \frac{\log(1+d)}{\log(1+r)} \cdot U^{\frac{\log(1+d)}{\log(1+r)}-1} \tag{6}$$

where, *U* is asset utilization level, calculated by D/C.

Whether (6) has extreme points or not with respect to *r* can be determined by calculating its derivative

$$\frac{d\left(\frac{\partial PV}{\partial D}\right)}{dr} = \frac{Asset}{C} \cdot U^{\frac{\log(1+d)}{\log(1+r)}} \cdot \left(-\frac{(\log(1+r))^{-2}}{(1+r)\cdot\ln 10}\right)$$
$$\cdot \left(1 + \ln U \cdot \frac{\log(1+d)}{\log(1+r)}\right) \tag{7}$$

Setting above formula to be equal to zero produces

$$\log(1+r) = -\ln U \cdot \log(1+d) \tag{8}$$

Rearranging (8) and solving it gives,

$$r = 10^{-\ln U \cdot \log(1+d)} - 1 \tag{9}$$

As both *U* and *d* are larger than 0, (9) is the solution of (8), defined as r^*. It indicates that (7) has an extreme point (maximum or minimum) with respect to *r*. This point must be included in building membership functions in the Vertex Extension [28].

III. FUZZY CHARGING METHODOLOGY

This section proposes a new charging model based on the concept of LRIC to accommodate fuzzy LGR. It consists of three subsections: III.A introduces the incremental LGR in α cut, III.B proposes a new charging function based on LRIC for computational efficiency, and III.C uses Vertex Extension approach to build charge membership functions.

A. Increment of Load Growth Rate

The fuzzy distribution of a LGR is supposed to be $\tilde{r}$ and a deterministic LGR related to one α–cut interval is r_d. The increment of LGR $\Delta\tilde{r}$ is

$$\Delta r = \tilde{r} - r_d \tag{10}$$

This increment reflects the derivation of r_d from the deterministic *r*. When a number of α–cut intervals are considered, the fuzzy LGR distribution $\tilde{r}$ can be discretised into a group of deterministic values.

B. Analytical Network Charging Function

In order to improve computational efficiency, an analytical charge calculating approach is introduced for incremental LGRs deviating from r_d. (4) can be linearised by considering the first term of its expansion in Taylor series around r_d

$$\Delta g(r) = \frac{\partial g(r)}{\partial r(r = r_d)} \cdot \Delta r \tag{11}$$

The above equation can be rewritten as

$$\Delta g(r) = \frac{\partial g(r)}{\partial n_{new}(r = r_d)} \cdot \Delta n_{new} + \frac{\partial g(r)}{\partial n(r = r_d)} \cdot \Delta n \qquad (12)$$

where, $\frac{\partial g(r)}{\partial n}$ is from

$$\frac{\partial g(r)}{\partial n} = AssetCost \cdot \frac{\ln(1+d)}{(1+d)^n} \qquad (13)$$

Taking the derivative of (4) with respect to n_{new} gives $\frac{\partial g(r)}{\partial n_{new}}$. Δn can be obtained by taking derivate of (2) with respect to r

$$\Delta n = -\frac{1}{1+r} \cdot \frac{\log C - \log D}{(\log(1+r))^2} \cdot \Delta r(r = r_d) \qquad (14)$$

Δn_{new} can be determined by replacing $\log D$ with $\log(D + \Delta p)$ in (14). Rearranging (11) and substituting above formula into it produces

$$\Delta g(r) = \frac{AssetCost \cdot \ln(1+d)}{(1+r) \cdot (\log(1+r))^2} \cdot$$
$$\left(\frac{\log C - \log(D + \Delta p)}{(1+d)^{n_{new}}} - \frac{\log C - \log D}{(1+d)^n} \right) \cdot \Delta r(r = r_d) \qquad (15)$$

It produces charges for an α–cut LGR deviating from r_d. For a small LGR increment Δr, it is easy to obtain incremental charge $\Delta g(r)$ with (15). Its ultimate charge is $\tilde{g}(r)$

$$\tilde{g}(r) = g(r)_d + \Delta g(r) \qquad (16)$$

where, $g(r)_d$ is the charge determined by using deterministic LGR. $\Delta g(r)$ is the incremental charge with respect to the LGR that deviates from r_d by using the incremental method.

C. Charge Membership Function

Here, the Vertex Extension proposed in [28] is used to build branch charge membership functions. It is based on the α-cut concept and interval analysis. The α-cut is a set of elements whose degree of membership is no less than alpha, used for setting fuzzy numbers [29]. First, the LRIC membership function is cut horizontally at a finite number of α-levels between 0 and 1. For each α-level, the model in approach in section III.B is used to determine incremental charges. This information is then used to construct membership functions. In the following steps, parameter I is α–cut charge value in response to different α-cut. The fuzz LGR model in Fig. 2 is employed to illustrate the implementation steps as follows:
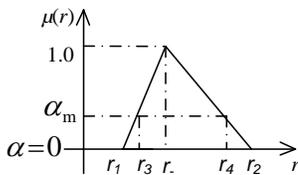

Figure.2 α–cut for fuzzy LGR model

i) Calculate the maximum charge for the loading condition with LGR $- r^*$ obtained in (9);
ii) When α=0, I_0=[r_1, r_2], calculate the charges by using the boundary LGRs- r_1 and r_2, and compare them

with the charge from step i). Here, I_{0+} is called as Support of the membership function;

$$B_{0+} = [\min(g(r_1), g(r_2), g(r^*)), \max(g(r_1), g(r_2), g(r^*))] \qquad (17)$$

iii) When α=α_m, $I_{\lambda m}$=[r_3, r_4], calculate charges with LGRs- r_3 and r_4, and compare them with that in step i);

$$B_{\alpha_m} = [\min(g(r_3), g(r_4), g(r^*)), \max(g(r_3), g(r_4), g(r^*))] \qquad (18)$$

iv) When α=1.0, I_1= [$\underline{r}$, $\underline{r}$], calculate charges with LGR- $\underline{r}$, and compare it with charges computed in step i). Here I_1 is called Core of the fuzzy membership function;

$$B_1 = [\min(g(\underline{r}), g(r^*)), \max(g(\underline{r}), g(r^*))] \qquad (19)$$

v) Repeat step iii) for other α -cut representations;
vi) Form charge membership functions using the obtained charges in steps (i)-(v).

The number of α–cuts is an arbitrage value [30] and in Vertex Method, the membership function is cut horizontally at a finite number of α-levels between 0 and 1. The α–cut number is determined not only by the preferable accuracy for charges, but also by the shape of fuzzy LGR models. Generally, the α–cuts that generate Core (when α=1) and Support (when α=0) of the fuzzy membership should be included. The selection of other α cuts is decided by related confidence level. As illustrated in [17], 5 α–cuts can generate results with good precision. The actual number is decided by network planners' judgement on load prediction, influenced by energy policy, economic development, etc.

IV. DEFUZZIFICATION OF CHARGE MEMBERSHIP FUNCTION

In practice, fuzzy charge membership functions need to be defuzzified to generate deterministic results. It can be realized by decision-making algorithms to generate the most appropriate deterministic values based on a given fuzzy set, such as Center of Area (COA), Center of Gravity (COG), Middle of Maximum (MOM), etc. [31]. Here, COG is adopted because of its advantages of having consistency, section invariance, monotonicity, linearity, and scale invariance [21]. COG finds the balance point by calculating the weighted mean of a fuzzy membership function, i.e. its gravity center. For a discrete membership function, the COG is

$$x_{COG}^* = \frac{\sum_{i \in V} x_i \mu_i(x)}{\sum_{i \in V} \mu_i(x)} \qquad (20)$$

where, x_i is the i-th domain point in the membership function, $\mu_i(x)$ is its confidence and V is the number of domain points.

V. IMPLEMENTATION STEPS

The implementation of the proposed approach for a single node is summarized in Fig. 3 with six key steps.
i) Form the fuzzy LGR model by using the method introduced in Section II;
ii) Map a fuzzy LGR model into the charging model by running the charging model with a group of selective α–cut intervals. Charges with other LGRs are evaluated with the proposed incremental method in Section III. More intervals are needed for more precise results;

iii) Form charge membership functions by using Vertex Extension approach [28].
- At light loading levels, charges increase with the rising LGR and the membership functions are formed directly by using the calculated charges;
- At heavy loading levels, charges decrease with rising LGR and membership functions can be built with the same process in the first case. The only difference is to reverse the coordinate direction of X axis;
- At medium loading levels, charges increase when LGR is small and then decrease when a summit is reached. The smaller charge between the two obtained from one LGR α–cut is chosen. The area beyond the summit point is excluded because that one charge can only have one confidence level;

iv) Defuzzify charge membership functions by using COG;

v) Calculate charge for one node. The summation of the defuzzified charges from all branches supporting it is its final nodal LRIC charge;

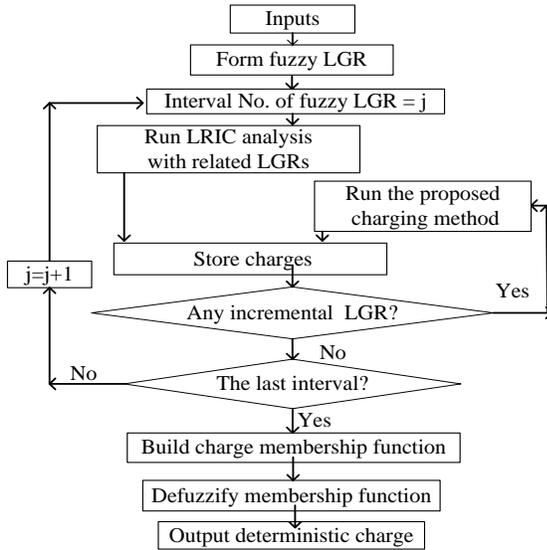vi) Charges for other nodes can be calculated by repeating steps ii) to v).

Fig.3. Flowchart of the proposed method

VI. CASE STUDY

This section demonstrates the proposed method on a two-busbar system and a practical UK distribution network. It is also compared with two existing approaches utilised by the UK industry- original LRIC and Investment Cost Related Pricing (ICRP) [32].

A. Demonstration on a Two-Busbar Test System

It is supposed that the simple two-busbar network in Fig. 4 has a single circuit with the capacity of 45MW after security redundancy and cost of £3,193,400. The combination of a typical discount rate of 6.9% [2] and 40-year lifespan produces an annuitized cost of £236,760. A quasi-triangular fuzzy LGR is modeled in Fig.5, where X axis represents the anticipated LGR and Y axis is confidence level. The LGR ranges from 1.4

% to 2.0% with a peak of 1.6% (confidence level 1.0) and is then discretized into 5 α–cut intervals in Figure 5.
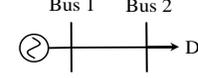

Fig.4. Two bus-bar test system

It is important to determine whether the charge member function with regard to LGR has extreme values. The relationship between the maximum/minimum network charges, circuit loading level and LGR calculated by (9) is graphically depicted in Fig.6. X axis is the circuit loading level and Y axis represents LGR. The combination of one loading level and LGR, i.e. a point on the line, produces the maximum/minimum charges. As seen, when LGR is big, the maximum/minimum charges appear at low loading level, vice versa. The charges at these points have to be included in charge membership functions because they determine the shapes.
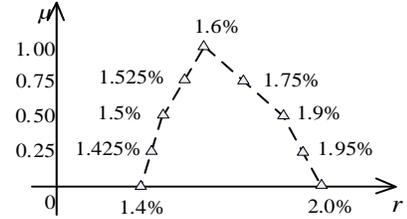
Fig.5. An unevenly distributed fuzzy load growth rate model
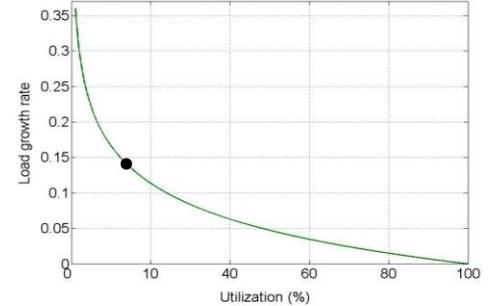
Fig.6. Variation of Load growth rate with loading level

TABLE I
CHARGES IN DIFFERENT LOADING LEVELS –PROPOSED APPROACH (£/MW/YR)

LGR	Loading level (MW)			
	20	30	35	40
1.4%	1274.1	5761	10255	16916
1.425%	1336.9	5847.6	10278	16768
1.45%	1400	5930.7	10297	16622
1.525%	1590.8	6159.8	10333	16191
1.6%	1782	6360.6	10343	15775
1.75%	2159.2	6687.2	10300	14987
1.9%	2521.3	6929	10196	14258
1.95%	2637.5	6993.5	10152	14028
2.0%	2751	7051	10103	13805

The demonstration is carried out at four circuit loading levels: 20MW (44%), 30MW (67%), 35M (78%) and 40MW (89%). The interim incremental costs related to each discretised LGR are given in Table I. The confidence level is the corresponding confidence level of each LGR α cut. As seen, 20 MW and 30 MW loading cases see continuously rising charges with respect to increasing LGR, but the 40MW

case has continuously declining charges. In 35MW loading case, charges keep on rising before a summit of 10340 £/MW/yr is reached at LGR of 1.75% and beyond this point, the charges decline steadily.

The above variations can be explained by (9), which determines charge inflection points with respect to LGR. For each loading level: 20MW (44%), 30MW (67%), 35MW (78%), and 40MW (89%), r^* are: 10.56%, 5.56%, 1.69%, and 0.79% respectively. The LGR used for building fuzzy model ranges from 1.4% - 2.0%. For 20MW and 30MW cases, their r^* are bigger than 2.0%, and thus their charges increase with increasing LGR. For 35MW case, 1.69% is within the range and thus the charges increase gradually and then drop when a summit is reached (LGR is around 1.69%). The 40MW case has r^* of 0.79%, which is smaller than 1.4% and, therefore, the charges decrease gradually with growing LGR.

TABLE II
CHARGES IN DIFFERENT LOADING LEVELS – ORIGINAL APPROACH (£/MW/YR)

LGR	Loading level (MW)			
	20	30	35	40
1.425%	1336.7	5849.4	10280	16768
1.525%	1589.9	6174.7	10349	16185
1.75%	2163.7	6735.6	10340	14957
1.95%	2638.7	6929.0	10154	14025

In order to testify the proposed analytical method for calculating charges, incremental branch costs calculated with LGRs of 0.25 and 0.75 confidence levels are produced in Table II. By comparing with the results in Table I (simulation results), the charges from the two approaches are very close in most cases. The biggest difference is 40 £/MW/yr (only 0.39%) for 35 MW loading case at the LGR of 1.75%. Such small difference is acceptable for the industry as Ofgem allows Distribution Network Operators (DNOs) to use both simulation and analytical approaches to derive incremental charges [15].

(a) 20MW loading case   (b) 30MW loading case

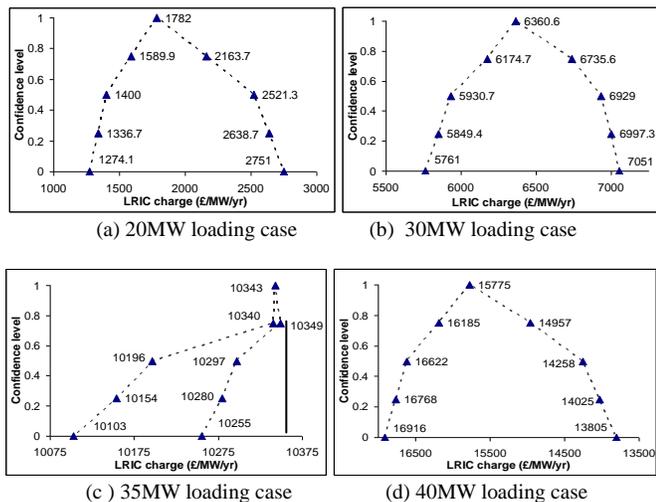(c) 35MW loading case   (d) 40MW loading case
Fig.7. Membership function of LRIC charges in different loading cases

By using charges in Table I, four membership functions in Fig.7 are built for the circuit under different loading levels.

Obviously, membership functions of 20MW and 30MW loading cases have very similar shapes, low LGRs seeing low charges and high LGRs producing high charges. The shape for 40MW loading case is similar to those of the two but charges decrease with increasing LGR. As seen in Fig.7.c, the shape for 35MW case is very different from those of others, charges increasing with rising LGR. A summit of 10349 £/MW/yr is reached, followed by a sharp decline, although LGR is still on the rise. Here, the highest charge dominates the shape between the area of 10225 £/MW/yr and 10349 £/MW/yr. The right-hand side boundary of the membership function should be redefined with the vertical solid line complying with the Vertex Extension approach.

The four charge membership functions are defuzzfied with COG to generate deterministic charges. The results together with those calculated with original LRIC approach (with a deterministic LGR of 1.6%) are given in Table III.

TABLE III
DETERMINISTIC CHARGE RESULTS FOR DIFFERENT LOADING CASES (£/MW/YR)

Method	Loading level			
	20MW	30MW	35MW	40MW
New fuzzy method	1962.6	6425.4	10257.8	15438.7
Original LRIC	1782.0	6360.6	10343.0	15775.0

Obviously, when the circuit is lightly loaded (20MW), the charge difference is big (180.6 £/MW/yr). This is because that at low loading level, charges increase rapidly with growing LGR and the defuzzified charge is from the area dominated by LGR higher than 1.6%. In 35MW case, although the maximum boundary of the membership function is dominated by 10349 £/MW/yr, the difference is relatively small around 100/£/MW/yr. For 40MW case, the charges are dominated by the area with LGR bigger than 1.6%, producing small charges. At low loading levels, charges from the proposed method are higher than those from original LRIC. It is opposite in higher loading cases because charge membership functions have inflection points with respect to LGR, illustrated in (9).

B. Demonstration on a Practical UK System

This section demonstrates the proposed method on a practical distribution area in Fig.8 taken from the UK network. This network has the voltage from 132kV to 33kV and its configuration includes both meshed and radial feeders. All parameters are from the local network operator.
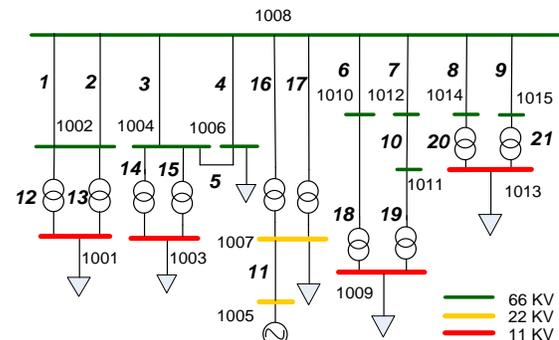

Fig.8. A practical test system in the UK from

This network includes all key features of typical Extra High Voltage (EHV) networks and thus is representative for demonstrating the proposed model. Its winter peak case is used to calculate charges. For simplicity, all loads are assumed to have the same fuzzy LGR membership function given in Fig.5. The discount rate and asset life spans are the same as those in the two-bus test system.

Table IV provides the charges of each branch to support certain nodes. The nodal incremental charges for node 1001 from the four supporting assets show different patterns: the costs from branches 1 and 2 are very small, merely 0.931 £/MA/yr, but the costs from transformers 12 and 13 are very high, reaching 1270 £/MA/yr. The reason is that the two circuits are lightly loaded at around 20%, but the two transformers are highly loaded at 43%. Another factor that leads to the big incremental costs is asset capital cost. The same reason applies to other branches that have varying incremental costs. One point worth noticing is that node 1006 has a negative incremental cost from branch 5. It means the demand receives a reward for using the branch, which is because any demand increment at node 1006 can reduce branch 4's power flow, thus delaying its investment.

It should be noted that not all branches supporting a node generate incremental charges, as charges are calculated according to the degree how their utilizations are affected by the new injection/withdrawal. If branch ratings are big, the impact of a nodal injection/withdrawal could be very small, thus hardly affecting their reinforcement horizons. Branches 6 and 10 supporting node 1009, and branches 8 and 9 supporting node 1013 all fall into this category.

TABLE IV
NODAL CHARGES CORRESPONDING TO RELATED BRANCHES (£/MW/YR)

Load	Branch No.				
1001	1	2	12	13	-
	0.93	0.93	1269.55	1270.89	-
1003	3	4	5	14	15
	2.34	1.75	0.19	694.06	696.30
1006	3	4	5	-	-
	2.19	1.82	-0.18	-	-
1007	16	17	-	-	-
	944.15	946.2	-	-	-
1009	7	18	19	-	-
	0.40	50.76	51.62	-	-
1013	20	21	-	-	-
	0.80	0.80	-	-	-

TABLE V
DETERMINISTIC CHARGE RESULTS FOR DIFFERENT LOADING CASES (£/MW/YR)

	Nodal charge					
	1001	1003	1006	1007	1009	1013
New fuzzy method	2542	1395	3.8	1890	103	1.6
Original LRIC	2777	1397	2	1896	82	1.0

Table V presents the charges for all nodes: the first row is obtained by the proposed method, and the second row

provides charges computed with the original LRIC with deterministic LGR model (1.6% for all nodes). The biggest difference is 234 £/MW/yr for node 1001. For nodes 1003 and 1007, the charges from the two approaches are very similar although they have big values. As for nodes 1006 and 1013, the differences grow up to nearly 50% of those from the deterministic model, but the absolute values are very small. The results illustrate the impact of uncertain LGR on network charges and the importance of modeling it by using a fuzzy model rather that a deterministic value.

The difference in charges from the original LRIC and the proposed method largely depends on the modeled LGR. If a relatively balanced fuzzy LGR is utilized, it is more likely to generate membership functions of balanced shape. The charges generated from the COG method are more likely to appear in the center of membership functions. However, if the derived fuzzy LGR model is not evenly distributed, the proposed method could produce fairly different charges compared with those from the original LRIC.

C. Comparison with Investment Cost Related Pricing

In this section, the new model is compared with the ICRP model used on UK transmission networks on the system in Fig. 7. The ICRP [32] respects not only the distance that electricity travels but also the degree of network utilization. Thus, it has been chosen by UK regulator- Ofgem as one standard methodology for calculating charges on transmission networks. ICRP calculates charges in system peak case. In this example, expansion factors and branch length used to derive charges are given in Table I. The expansion constant is chosen as £9.24/MW×km×year [24] and the resultant charges are given in Table VII.

TABLE VI
SYSTEM PARAMETERS FOR ICRP METHOD

Branch No.	From bus	To bus	Length (km)	Expansion Factor
1	2	8	6.131	2.8
2	4	8	11.3	2.8
3	6	8	9.079	2.8
4	6	4	1.988	2.8
5	8	2	6.164	2.8
6	8	10	10.706	2.8
7	8	12	13.24	2.8
8	12	11	2.736	2.8
9	14	8	3.661	2.8
10	15	8	7.137	2.8
11	7	5	0	1.14
12	2	1	28.75	1
13	2	1	28.75	1
14	4	3	40	1
15	4	3	40	1
16	8	7	31.25	1.14
17	8	7	31.25	1.14
18	10	9	40	1
19	11	9	40	1
20	14	13	50	1
21	15	13	50	1

TABLE VII
NODAL CHARGES FOR DEMAND UNDER ICRP (£/MW/YEAR)

Node	1001	1003	1006	1007	1009	1013
Charge	764	1186	428	592	1286	1083

By comparing the charges in Tables V and VII, it is seen that they vary dramatically. Particularly for node 1013, the first two methods (fuzzy LRIC and original LRIC) produce fairly small value 1.6£/MW/yr and 1.0£/MW/yr respectively, while ICRP generates very high -1083£/MW/yr. The big difference is due to the different rational of approaches. ICRP assumes that any additional power injection requires immediate network investment, thus not able to recognize the spare capacity in networks. The same reason applies to the charge difference at other nodes.

VII. PRACTICAL CONSIDERATIONS

The use of regional growth rates might to some extent reduce the impact of uncertain demand in network pricing, but challenges still exist: i) it is hard to determine how big area of networks can use the same region LGRs; ii) it would increase the calculation and management burden for operators to calculate regional LGRs and reflect them in charging.

This new method is a good candidate for network pricing when uncertain load growth exists. All data used for deriving network prices includes network topology, equipment type, costs, etc. are from network operators [33]. This fuzzy LGR model can be actually derived from network operators' load forecasting, because in order to identify needed investment, they also carry out load forecasting with different scenarios. As the owner and operators of distribution networks, they have all the information, as the data underpins for network planning, operation and control conducted by network operators. Other key parameters, such as interest rate, discount rate, annuity factor, are set or calculated by regulatory authorities [33]. Practically, the regulator reviews DNOs' load growth projection to ensure that it complies with national economic development and networks are well developed to accommodate increasing generation and demand.

We have compared the proposed approach with two existing standard approaches utilised by the UK industry- LRIC and ICRP. Results show that the new approach can generate reasonable charges close to those from the original LRIC in light loading conditions, but very different in heavy loading cases. By contrast, due to different principles, the new approach and ICRP produce very different results. The contribution of this paper is not to improve the precision of charge calculation but to expand the capability of existing charging approaches to include uncertain load growth. Into the future, if DNOs have more and accurate information on peak load growth, such as from smart metering, probabilistic pricing models would be possible to develop.

Practically, distribution networks are still a bit dummy compared to transmission networks and physically, they are generally radial, with some parts meshed. The generation size that distribution networks accommodate is relatively small but large in amount. Most domestic and commercial customers are connected here, inducing a large amount of low-carbon technologies, such as EVs, PVs, in the future. Transmission networks are for large-scale and long-distance power transfer, and by contrast distribution networks are for distributing the power from transmission networks to different customers across wide areas and accommodating small-size generation. Considering their distinctively different structures and functionalities, network pricing should be designed for them respectively.

If network pricing needs to be close to real-time, the non-linear characteristics of devices have to be included. Our approach assumes that each load growth projection has a relevant confidence and network planners have the rough knowledge of its range according to their knowledge. In the future, our model will be extended to lower voltage networks with limited information and improved by including more smart devices through dynamic modeling.

VIII. CONCLUSIONS

This paper presents a novel long-run network charging method to include uncertain load growth rate. The method models future load growth as a fuzzy membership function and maps it into a new analytical charging method with the Vertex Extension method. The demonstration examples on two systems provide the following observations:

i) The new fuzzy load growth rate model can reflect the uncertainties in demand growth of different confidence levels. It can be easily mapped into the new charging model by using a set of α–cut intervals;

ii) The proposed analytical charging model is effective in deriving network charges with fuzzy LGR. Its major advantage is high computational efficiency as there is no need to rerun the whole charge calculating process for all α–cut intervals;

iii) The charge difference between the proposed method, the original deterministic LRIC, and ICRP vary, the degree of which is decided by the shapes of used fuzzy load growth rates. With uneven fuzzy load growth rate, big difference is expected, indicating that fuzzy load growth rates should be properly modeled.

The new charging approach is useful for network operators to price network users with uncertain load growth. Future research will look at the possibility of using probabilistic models to model the uncertainties in load growth.

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