Quantification of Additional Reinforcement Cost from Severe 3-Phase Imbalance

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Abstract—**This letter is an enhancement to our previous paper that quantifies additional reinforcement costs (ARCs) for low voltage (LV) assets under moderate degree of 3-phase imbalance. The original formulas cause an overestimation of the ARCs under severe imbalance. This letter first quantifies the threshold of the severe degree of imbalance (DIB), below which the original formulas are applicable. Then, the ARC formulas are extended to account for the whole range of DIB. Case studies demonstrate that when the asset loading level is below 33.3% (50%) for a feeder (a transformer), the DIB never exceeds the threshold and the original ARC formulas are applicable; otherwise, the DIB can exceed the threshold and the extended formulas yield correct ARCs.**

Index Terms—**Distribution network investment, three-phase imbalance, low voltage**

I. Introduction

THREE-PHASE imbalance causes additional reinforcement costs (ARCs) beyond the reinforcement costs for the balanced case because: for low voltage (LV) main feeders (referred to as 'feeders'), it reduces the available capacity as the phase with the least margin determines the available capacity; for LV transformers (referred to as 'transformers', i.e. the typical 11kV/415V Delta-Wye-n transformers), it reduces the available capacity by inducing a power flow along the neutral wire [1]. When facing the three-phase imbalance issue without customers' phase connectivity data, it is common for distribution network operators (DNOs) to reinforce the network in a conventional way (e.g. investing in new lines and transformers) when the asset capacity is reached, i.e. a passive option [2]. As a key cost element, the ARC has to be quantified for DNOs to appraise the passive option. In future, with increasing knowledge of customers' phase connectivity, it is possible to use alternative 'smart' options for short-term phase balancing. The ARC quantified in this letter will therefore serve as a benchmark cost, with which the cost of alternative solutions can be compared.

We previously published a paper on the quantification of the ARC from three-phase imbalance for feeders and transformers [1]. The previous ARC formulas work only under a moderate degree of three-phase imbalance. A time horizon is defined as the number of years for the peak demand to reach the thermal limits of the asset under long-term demand growth. Under a severe degree of imbalance (DIB), the previous formulas lead to negative time horizons and overestimated ARCs. The border

between a moderate DIB and a severe DIB is the threshold of the severe DIB. The novelty of this letter lies in the quantification of this threshold and the extended ARC formulas that work under the full range of DIB, including the severe DIB case. The methodology applies to European style three-phase four-wire low voltage networks.

II. Extended ARC Formula for Feeders

A time horizon is defined in Section I. The original formula for calculating the time horizon of a feeder under 3-phase imbalance was [1]:

$$n_{\text{T\_IB}} = -\frac{\log U_{\text{N}} + \log(3D_{\text{IB}} + 1)}{\log(1 + r)} \tag{1}$$

where U_{N} denotes the nominal utilization rate as the three-phase peak power over the rated capacity ($0 < U_{\text{N}} \leq 1$); D_{IB} and r denote the feeder's DIB as defined in [1] and the annual demand growth rate, respectively.

The natural boundary of the DIB is $0 \leq D_{\text{IB}} \leq 2/3$ [1]. The threshold of severe DIB is found by solving the equation $n_{\text{T\_IB}} = 0$, subject to the above limit:

$$D_{\text{IB\_thre}} = \min\{\frac{1}{3U_{\text{N}}} - \frac{1}{3}, \frac{2}{3}\} \tag{2}$$

It should be noted that $U_{\text{N}} \in [0, 100\%]$.

The original feeders' ARC formula in [1] only works when $D_{\text{IB}} \leq D_{\text{IB\_thre}}$. When $D_{\text{IB}} > D_{\text{IB\_thre}}$, the original $n_{\text{T\_IB}}$ equation in [1] yields a negative $n_{\text{T\_IB}}$. Equation (9) in [1] then calculates the reinforcement cost based on the negative $n_{\text{T\_IB}}$ – this makes no sense in reality because network reinforcements can only take place no earlier than now. Consequently, equation (10) in [1] overestimates the ARC when $D_{\text{IB}} > D_{\text{IB\_thre}}$.

When $D_{\text{IB}} = D_{\text{IB\_thre}}$, the phase imbalance becomes so severe that the peak demand of the 'heaviest' phase reaches the thermal capacity of that phase (corresponding to $n_{\text{T\_IB}} = 0$) – the distribution network operator (DNO) needs to make network reinforcements immediately [1]. When $D_{\text{IB}} > D_{\text{IB\_thre}}$, the peak demand exceeds the thermal capacity of the 'heaviest' phase, in which case this letter keeps $n_{\text{T\_IB}}$ at zero rather than letting it go negative, thus avoiding an overestimation of the ARC: this is justified because the network reinforcements can only take place no earlier than now.

When $U_{\text{N}} \geq 1/3$, $D_{\text{IB\_thre}}$ decreases with the increase of U_{N} – a greater nominal utilization corresponds to a lower tolerance on three-phase imbalance; when $U_{\text{N}} < 1/3$, there is $D_{\text{IB\_thre}} = 2/3$ and it is certain that $D_{\text{IB}} \leq D_{\text{IB\_thre}}$ and $n_{\text{T\_IB}} > 0$.

The time horizon formula is updated as:

$$n_{\text{T\_IB}} = \begin{cases} -\dfrac{\log U_{\text{N}} + \log(3D_{\text{IB}} + 1)}{\log(1 + r)} & \text{when } D_{\text{IB}} < D_{\text{IB\_thre}} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The new ARC formula for feeders is extended to account for the full range of DIB:

$$\Delta PV_{3\emptyset\text{IB}} =$$
$$\begin{cases} Asset_I(1+d)^{\frac{\log U_{\text{N}}}{\log(1+r)}}\left[(1+d)^{\frac{\log(3D_{\text{IB}}+1)}{\log(1+r)}} - 1\right] & \text{when } D_{\text{IB}} < D_{\text{IB\_thre}} \\ Asset_I\left[1 - (1+d)^{\frac{\log U_{\text{N}}}{\log(1+r)}}\right] & \text{when } D_{\text{IB}} \ge D_{\text{IB\_thre}} \end{cases} \quad (4)$$

where $Asset_I$ and d denote the future investment cost and the discount rate, respectively.

III. EXTENDED ARC FORMULA FOR TRANSFORMERS

Similar to feeders, the original formula for calculating the time horizon $n_{\text{T\_IB}}$ of a transformer under imbalance was equation (19) in [1]. The threshold of severe DIB, $D_{\text{IB\_thre\_T}}$, is obtained by solving the equation $n_{\text{T\_IB}} = 0$, subject to $D_{\text{IB\_thre\_T}} \in [0, 1.0]$.

$$D_{\text{IB\_thre\_T}} = \min\{\frac{1}{U_{\text{N}}} - 1, 1\} \quad (5)$$

When $D_{\text{IB\_T}} \ge D_{\text{IB\_thre\_T}}$, the degree of imbalance is severe enough that a sizable neutral line power causes the thermal capacity of the transformer to be reached. In this case, this letter keeps the time horizon $n_{\text{T\_IB}}$ at zero rather than letting it go negative, thus avoiding an overestimation of the ARC.

When $U_{\text{N}} \ge 1/2$, $D_{\text{IB\_thre\_T}}$ decreases with the increase of U_{N} – a greater nominal utilization corresponds to a lower tolerance on three-phase imbalance; when $U_{\text{N}} < 1/2$, $D_{\text{IB\_thre\_T}} = 1$, in which case $D_{\text{IB\_T}}$ never exceeds the threshold (i.e. $D_{\text{IB\_T}} \le D_{\text{IB\_thre\_T}}$) and $n_{\text{T\_IB}} > 0$.

The time horizon formula is extended as:

$$n_{\text{T\_IB}} = \begin{cases} -\dfrac{\log U_{\text{N}} + \log\left(D_{\text{IB\_T}} + 1\right)}{\log(1 + r)} & \text{when } D_{\text{IB\_T}} < D_{\text{IB\_thre\_T}} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

The new ARC formula applicable for the full range of DIB for transformers is extended from [1]:

$$\Delta PV_{3\emptyset\text{IB}} =$$
$$\begin{cases} Asset_I(1+d)^{\frac{\log U_{\text{N}}}{\log(1+r)}}\left[(1+d)^{\frac{\log(D_{\text{IB\_T}}+1)}{\log(1+r)}} - 1\right] & \text{when } D_{\text{IB\_T}} < D_{\text{IB\_thre\_T}} \\ Asset_I\left[1 - (1+d)^{\frac{\log U_{\text{N}}}{\log(1+r)}}\right] & \text{when } D_{\text{IB\_T}} \ge D_{\text{IB\_thre\_T}} \end{cases} \quad (7)$$

where $Asset_I$ and d denote the future investment cost and the discount rate, respectively. Other variables are defined the same as in previous formulas.

IV. CASE STUDY

All input data are field data from Network Design Manual published by the DNO [3]. The future reinforcement costs of a feeder (e.g. a suburban cable of 300m) and a transformer (e.g. an urban one) are £13,440 and £26,400, respectively [3]. For the feeder, Fig. 1 proves the relation between $D_{\text{IB\_thre}}$ and U_{N} to be consistent with the statement in Section II. Fig. 2 demonstrates that when $U_{\text{N}} = 60\% (> 33.3\%)$, the ARC increases with D_{IB} until reaching a ceiling when $D_{\text{IB}} \ge D_{\text{IB\_thre}}$ – the imbalance is too severe and network reinforcements need

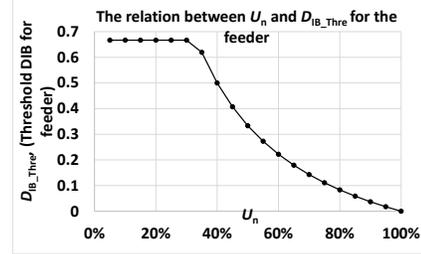to take place now; when $U_{\text{N}} = 30\%$, the ARC increases with D_{IB} which never exceeds $D_{\text{IB\_thre}}$.

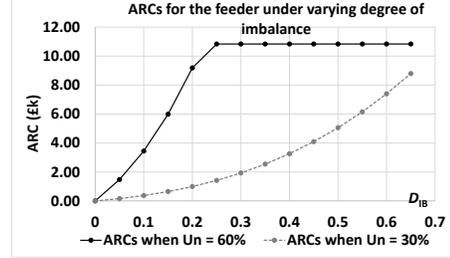Fig.1. The relation between $D_{\text{IB\_Thre}}$ and U_{N} for the feeder

Fig.2. ARCs for the feeder under varying D_{IB}

For the transformer: Fig. 3 proves the statement about the relation between $D_{\text{IB\_thre\_T}}$ and U_{N} in Section III. Fig. 4 demonstrates that when $U_{\text{N}} = 80\% (> 50\%)$, the ARC increases with $D_{\text{IB\_T}}$ until reaching a ceiling when $D_{\text{IB\_T}} \ge D_{\text{IB\_thre\_T}}$ – the imbalance is too severe and network reinforcements need to take place now; when $U_{\text{N}} = 40\%$, the ARC increases with $D_{\text{IB\_T}}$ which never exceeds $D_{\text{IB\_thre\_T}}$.
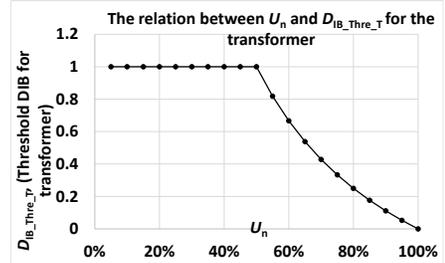

Fig.3. The relation between $D_{\text{IB\_Thre\_T}}$ and U_{N} for the transformer
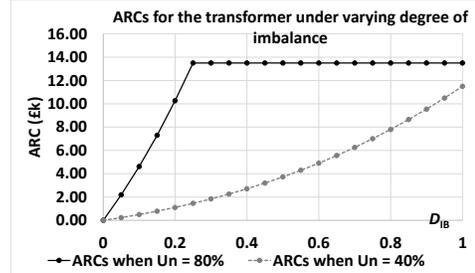

Fig.4. ARCs for the transformer under varying $D_{\text{IB\_T}}$

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